

SEMESTER-V

COURSE 15: NUMBER THEORY

Theory

Credits: 4

5 hrs/week

Learning Outcomes

After successful completion of the course, students will be able to

1. understand the fundamental theorem of arithmetic
2. understand Mobius function, Euler quotient function, The Mangoldt function, Liouville's function, The divisor functions and the generalized convolutions.
3. understand Euler's summation formula, application to the distribution of lattice points and the applications to $\mu(n)$ and $\Lambda(n)$
4. understand the concepts of congruencies, residue classes and complete residues systems.
5. Comprehend the concept of quadratic residues mod p and quadratic non residues mod p .

UNIT-I

The Fundamental Theorem of Arithmetic

Introduction, Divisibility, Greatest common divisor, Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes, The Euclidean algorithm, The greatest common divisor of more than two numbers

UNIT-II

Arithmetical Functions And Dirichlet Multiplication

Introduction- The Mobius function $\mu(n)$ – The Euler quotient function $\varphi(n)$ - A relation connecting φ and μ - A product formula for $\varphi(n)$ - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The Mangoldt function $\Lambda(n)$ - multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function $\lambda(n)$ - The divisor functions $\sigma_\alpha(n)$

UNIT-III

Averages Of Arithmetical Functions

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summation formula- Some elementary asymptotic formulas-The average order of $d(n)$ - The average order of the divisor functions $\sigma_\alpha(n)$ - The average order of $\varphi(n)$ - An application to the distribution of lattice points visible from the origin- The average order of $\mu(n)$ and $\Lambda(n)$ -The partial sums of a Dirichlet product- Applications to $\mu(n)$ and $\Lambda(n)$

UNIT-IV

Congruences

Definition and basic properties of congruences- Residue classes and complete residue systems- Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo p . Lagrange's theorem- Applications of Lagrange's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applications of the Chinese remainder theorem

UNIT-V

Quadratic Residues and the Quadratic Reciprocity Law

Quadratic Residues, Legendre's symbol and its properties, Evaluation of $(-1/p)$ and $(2/p)$, Gauss lemma, The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol, Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums, Another proof of the quadratic reciprocity law.

Activities

Seminar/ Quiz/ Assignments/ Applications of Number theory to Real life Problem /Problem Solving Sessions

Text Book

Introduction to Analytic Number Theory by T.M.Apostol, Springer Verlag-New York, Heidelberg-Berlin-1976.

Reference Books

1. Elementary Number Theory by G.A.Jones and J.M.Jones, , Springer
2. Elementary Number Theory by David, M. Burton, 2nd Edition UBS Publishers.
3. Number Theory by Hardy & Wright, Oxford Univ., Press.
4. Elements of the Theory of Numbers by Dence, J. B &Dence T.P, Academic Press
